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Topography optimization of the vibrating structure for fused deposition modelling of parts considering a hybrid deposition path pattern

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ABSTRACT

Frequency optimization plays a vital role in designing machines and structures to avoid destructive responses caused by external excitation. In the current study, most frequency optimization research focuses on algorithm innovation to pursue better numerical results. However, with the development of additive manufacturing, increasingly more organic structures produced by topology optimization can be physically fabricated. Therefore, the combination of topology optimization and additive manufacturing is promising and widely investigated. This paper proposes a concurrent topology optimization method for maximizing the natural frequency of Fused Deposition Modelling (FDM) parts printed by a Hybrid Deposition Path (HDP) pattern. The proposed algorithm concurrently optimizes the shape of the structure and the raster directions of the substrate domain, wherein the method of solid orthotropic materials with penalization (SOMP) with double layers of smoothing and projection (DSP) is adopted. A dedicated sensitivity analysis is performed on both the topology and direction variables. Several numerical results are studied to show the effectiveness of the proposed method is efficient and to disclose the influence of raster direction on the vibration model. This work would be instructive to design for FDM printing.

1. Introduction

Within the DfAM (design for additive manufacturing) framework, topology optimization has been treated as a dominating structure design method (J. Liu et al. 2018; Wei et al. 2015; Brackett, Ashcroft, and Hague 2011; Huang et al. 2020). It seeks optimal material distribution within a given domain subject to predefined physical conditions. Various topology optimization methods, e.g., the homogenization method (Martin Philip Bendsoe and Kikuchi 1988), solid isotropic material with penalization (SIMP) (Xie et al. 2021b) evolutionary structural optimization (ESO) (Xie and Steven 1997) and level-set method (Sethian and Wiegmann 2000; M.Y. Wang, Wang, and Guo 2003), etc. have been proposed to address the multidisciplinary engineering optimization problems. On the other hand, tough to manufacture is a severe limitation for topology optimization since it tends to output binary models representing complex geometry, raising up the tool accessibility issue, demolding issue, over-thin structural member issue, etc. Tremendous efforts have been put into developing and solving manufacturability constraints, while the compromise on optimized structural performance seems unavoidable. In recent years, the emergence of additive manufacturing (AM) technique brings up the opportunity of fully addressing the manufacturability issues. Additive manufacturing, unlike conventional manufacturing processes, e.g., milling, turning, boring, etc., that remove materials from the raw part, deposits materials layer-by-layer and builds the part in a novel bottom-to-top manner (Gibson et al. 2021). AM fills the gap between design and manufacturing due to this unprecedented processing approach and brings in the most significant design freedom ever. Hence, design for additive manufacturing (DfAM) through topology optimization has emerged as a research hotspot in recent years (Rosen 2007; Ponche et al. 2014; Huang et al. 2021), and a number of dedicated topology optimization for AM methods have been developed and widely recognized (J. Liu and To 2017; XU, Liu, Huang et al. 2021a; Liu and Huangchao 2020; J. Liu et al. 2022; Shuzhi, Liu, and Yongsheng 2022).

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As one of the main branches of topology optimization, frequency optimization has been studied in many cases, which is desirable to shift the natural structure frequency to avoid destructive response caused by external excitation. Also, high natural frequency structures tend to be reasonably stiff for all conceivable loads (Bendsoe and Sigmund 2003). The first work on natural frequency optimization can be traced back to Diaaz and Kikuchi (1992), who maximized the fundamental frequency of a plate under a maximum volume constraint. Following this, numerous researches on frequency optimization have been carried out. Du and Olhoff (2007) developed a general solution procedure that allows for eigenfrequency multiplicity. Zheng-Dong, Cheng, and Kikuchi (1994) presented the structural designs accessing desired eigenfrequencies through concurrent topology and shape optimization. Zhang, Gao, and Xiao (2020a) proposed a Kriging-assisted multi-scale topology optimization method for maximizing natural frequencies of inhomogeneous cellular structures. Interested readers can refer to Zaraghan’s work (2016) for a compressive study on topology optimization for natural frequency problems. To date, most of the researches on natural frequency optimization problems focus on algorithm innovation to pursue better numerical results, while topology optimization targeting the natural frequency properties of AM structures is rarely addressed, especially given the scope of concurrent structure and process setup optimization.

As widely recognized, AM produces anisotropic material properties (P. Zhang, Liu, and To 2017), since a layer-by-layer deposition process forms the model. The tensile modulus and strength in the raster direction, the transverse direction, and the build direction tend to be different, especially for the fused deposition modeling (FDM) process (Mahmood et al. 2016; Huangchao et al. 2020; Lopez, Moises, and Ahmad 2020). This anisotropic property was often ignored in computational designs (Rezaie et al. 2013; Baca and Ahmad 2020), while performing topology optimization addressing the anisotropic material properties derives optimized designs closer to reality (J. Liu 2016; J. Liu and Huangchao 2017). Moreover, due to the direction variables are introduced into the optimal process, enhanced design space can be accessed. Most fused deposition modeling (FDM) machines work in the hybrid deposition path (HDP) pattern, as depicted in Figure 1, the structural contour is offset for a distance for contour-offset deposition and unidirectional zigzag paths fill the interior with the parallel deposition directions. Some topology optimization researchers have taken the HDP-related material anisotropy into account. Jikai et al. (2018) proposed a level set-based method that integrates optimal hybrid deposition paths with shape and topology optimization. A recent algorithm was developed by Shuzhi et al. (2020) to find out an optimal structure design that considers the HDP pattern and the anisotropic material properties. However, natural frequency-oriented topology optimization taking into account the HDP pattern is not addressed yet.

The difficulty of realizing topology optimization considering the HDP pattern lies in distinguishing between the contour-offset layer and the substrate domain. In this work, the double smoothing and projection (DSP) approach is used to address the above challenge. This DSP method was first proposed by Clausen, Aage, and Sigmund (2015) to design the coated structures that enhance the base structure with a strong uniform-thickness solid coating. It was verified that the DSP method could produce a uniform interface and control the thickness of the interface by changing the second projection radius. This method was applied and further developed by Luo, Quhao, and Liu (2019), Yoon and Bing (2019), and Wang and Kang (2018). In addition, to consider the anisotropic material properties, the classic Solid Isotropic Material with Penalization (SIMP) interpolation method is not suitable, which is mainly used for the interpolation of isotropic materials (Zhou and Rozvany 1991; Martin P Bendsoe and Sigmund 1999). In contrast, the solid orthotropic material with penalization (SOMP) is adopted in this essay.
To enlarge the design space, in addition to the density design variables, the raster directions in the substrate domain are included as another design variable that controls the material anisotropy. Note that this direction variable is a single variable that does not differ element-wise. For FDM machines, the zigzag print paths of the substrate domain follow two opposite but parallel directions. Finally, both the density and direction variables are concurrent updates by the Method of Moving Asymptote (MMA) algorithm (Svanberg 1987).

For concurrent topology optimization, Gao et al. 2019a; Gao et al. 2019b presented a set of concurrent topology optimization for composite structure design. As to frequency optimization of concurrent topology optimization, Yan et al. (Y. Zhang et al. 2020b) proposed an efficient concurrent topology optimization method for minimizing the frequency response of cellular composites over a given frequency interval. However, these aforementioned works focus on multiscale designs, the concurrent topology optimization combining with printing parameters, such as printing angle is still in lacking.

In conclusion, this work features in solving the natural frequency topology optimization problem incorporating HDP pattern-related material anisotropy. A new algorithm based on DSP and SOMP, concurrently optimizing the density and raster direction variables, is proposed to generate optimal structural solutions incorporating the material anisotropy effect. The rest of the content is structured as follows: Problem definition is presented in section 2, including the optimization model and the corresponding interpolation scheme. Section 3 presents the sensitivity analysis. Several numerical results are studied and discussed in Section 4. Finally, Section 5 concludes the paper.

2. Problem definition

The first part of this section briefly introduces the state equation for frequency analysis. Then, a topology optimization algorithm considering the substrate domain’s HDP pattern and raster direction is proposed. The material model and characteristic properties are derived analytically based on continuous versions of the design field in the third subsection. The optimization model is formulated in the end.

2.1 State equation for frequency analysis

In general, the topology optimization problem for maximum nature frequency considers the undamped free vibration situation. The relationship between damped nature frequency \( \omega_d \) and undamped nature frequency \( \omega_n \) is shown as below:

\[
\omega_d = \omega_n \sqrt{1 - \xi^2}
\]

where \( \xi \) is the damping ratio. It is noted that most structures’ damping ratio is below 10%, which shows that damping only has limited influence on natural frequency and is thus usually ignored in the optimization model. The dynamic behavior of a free vibration structure without damping can be written in the matrix form as:

\[
(K - \omega_k^2 M)u_k = 0
\]

where \( \omega_k \) represents the \( k \)th natural frequency. In general, the fundamental frequency refers to the lowest nature frequency \( \omega_1 \). \( u_k \) is the eigenvector corresponding to \( \omega_k \) and represents the displacement of vibration. \( K \) and \( M \) denote the global stiffness and mass matrix of the structure, respectively. They can be calculated as follows:

\[
M = \sum_{e=1}^{NE} \int_{\Omega_e} N^T \rho_e N d\Omega, K = \sum_{e=1}^{NE} \int_{\Omega_e} B^T D_e B d\Omega
\]

where \( B \) and \( N \) denote the strain-displacement matrix and shape function matrix, respectively. \( NE \) represents the total number of elements in the design domain. \( \Omega_e \) represents the domain of the \( e \)th element. \( \rho_e \) and \( D_e \) mean the density and elasticity tensor of the solid element whose detailed expression will be derived in the latter content.

Based on the Rayleigh quotient, the natural frequency \( \omega_k \) can be written as

\[
\omega_k^2 = \frac{u_k^T K u_k}{u_k^T M u_k}
\]

Usually, the eigenvectors are mass-matrix-normalized:

\[
u_k^T M u_k = 1
\]

2.2 Topology optimization algorithm

The density design variables could explicitly represent the HDP-based AM process by adopting a series of filtering, projection, and gradient norm operations.
This procedure is achieved by the extension of the DSP method. As illustrated in Figure 2, the first density field $\mu$, which denotes the element densities, is smoothed into $\tilde{\mu}$ to get rid of checkerboard patterns. Then, the smoothed field $\tilde{\mu}$ is projected to create $\varphi$, representing the physical densities with sharp boundaries. To generate the contour-offset area, the second smoothing filter is applied to the field $\varphi$ to obtain the smoothed density field $\hat{\varphi}$, and the interface between the base region and voids is identified by taking gradient norms on $\hat{\varphi}$, as:

$$\nabla \hat{\varphi}_a = a \nabla \hat{\varphi} = a \sqrt{\left( \frac{\partial \hat{\varphi}}{\partial x} \right)^2 + \left( \frac{\partial \hat{\varphi}}{\partial y} \right)^2}$$

wherein $a$ is a normalization factor defined as the inverse of the maximum possible gradient norm of $\hat{\varphi}$. The relationship between, smoothing radius $R_2$, and interface layer thickness $t$ shows below [33]:

$$a = \frac{R_2}{\sqrt{3}} \approx \frac{2.5t}{\sqrt{3}}$$

The normalized gradient norm $\nabla \hat{\varphi}_a$ is subsequently manipulated with Heaviside projection with parameters $\beta_2$ and $n_2$ to define clearly the contour-offset area $\tau$. The raster directions for contour offset area are calculated based on $\hat{\varphi}$ by

$$\theta_\tau = \frac{\pi}{2} + \arctan\left( \frac{\partial \hat{\varphi}}{\partial x} / \frac{\partial \hat{\varphi}}{\partial y} \right)$$

In this work, all the orientations are counted in the counter-clockwise direction, as shown in Figure 3.

The relationship among the variables is depicted in Figure 2. The variable $\theta_\varphi$ for the raster direction of the substrate domain and the contour profiles directions $\theta_t$ are introduced to the substrate domain $\varphi$ and the contour-offset layer $\tau$, respectively, to define the anisotropic material properties.
2.3 Material interpolation

Based on the SOMP model, the $e^{th}$ element's elasticity tensor $D_e$ is interpolated with the element densities $\varphi_e$ and $\tau_e$:

$$D_e(\varphi_e, \tau_e) = (\varphi_e)^P D_\varphi + (\tau_e)^P D_T - (\varphi_e)^P (\tau_e)^P D_\varphi$$  \hspace{1cm} (9)

where $P$ is employed to penalize the intermediate densities; $D_\varphi$ and $D_T$ are the elasticity tensors of the anisotropic materials for the substrate and contour-offset areas, respectively. According to the classic laminate theory, the elasticity tensor containing the orientation variable is formulated in Eq. (10),

$$D(\theta) = T(\theta) D_0 T(\theta)^T$$  \hspace{1cm} (10)

where $D_0$ is the laminate unrotated stiffness tensor, and $T$ is the coordinate transformation matrix.

$$D_0 = \begin{bmatrix}
\frac{E_x}{1-v_{yx}v_{yx}} & \frac{v_{yx}E_y}{1-v_{yx}v_{yx}} & 0 \\
\frac{v_{yx}E_x}{1-v_{yx}v_{yx}} & \frac{E_y}{1-v_{yx}v_{yx}} & 0 \\
0 & 0 & G_{xy}
\end{bmatrix}$$  \hspace{1cm} (11)

$$T(\theta) = \begin{bmatrix}
\cos^2\theta & \sin^2\theta & -2\sin\theta\cos\theta \\
\sin^2\theta & \cos^2\theta & 2\sin\theta\cos\theta \\
\sin\theta\cos\theta & -\cos\theta & \cos^2\theta - \sin^2\theta
\end{bmatrix}$$  \hspace{1cm} (12)

The density interpolation could be obtained in a similar way,

$$\rho_e(\varphi_e, \tau_e) = m_\varphi \varphi_e + m_\tau (1 - m_\varphi \varphi_e) \tau_e$$  \hspace{1cm} (13)

where $m_\varphi$ is the mass density of the substrate material, $m_\tau$ is the mass density of the contour-offset area material. Extreme cases can verify this interpolation. For the contour field, $\tau_e = 1$ and $\varphi_e = 0$, then:

$$\rho_e(0, 1) = m_\tau$$  \hspace{1cm} (14)

$$D_e(0, 1) = D_T$$  \hspace{1cm} (15)

For substrate field, $\tau_e = 0$, then:

$$\rho_e(\varphi_e, 0) = m_\varphi \varphi_e$$  \hspace{1cm} (16)

$$D_e(\varphi_e, 0) = (\varphi_e)^P D_\varphi$$  \hspace{1cm} (17)

2.4 Smoothing and projection

In the DSP method, there are two smoothing filters, and two projection filters applied. A so-called PDE filter based on the Helmholtz-type partial differential equation (Lazarov and Sigmund 2011) is applied for smoothing. The smoothed density field $\hat{\nu}$ of density field $\nu$ is implicitly defined as a solution to the Helmholtz PDE:

$$- r^2 \nabla^2 \hat{\nu} + \hat{\nu} = \nu, \nu \in \{\mu, \varphi\}$$  \hspace{1cm} (18)

where $r$ is a length scale parameter, its value is determined by the filter radius, $R$. 

$$r = \frac{R}{2\sqrt{3}}$$  \hspace{1cm} (19)

After each smoothing filter, the density value falls between 0 and 1. Then, the Heaviside projection filter is applied to obtain binary density 0 or 1 to obtain the black-and-white design. The relationship between projected value $\hat{\nu}_i$ and smoothed density value $\hat{\nu}_i$ is demonstrated below (F. Wang, Lazarov and Sigmund 2011).

$$\hat{\nu}_i = \frac{\tanh(\beta \eta) + \tanh(\beta (\hat{\nu}_i - \eta))}{\tanh(\beta \eta) + \tanh(\beta (1 - \eta))}, \hat{\nu}_i \in \{\hat{\mu}, \hat{\varphi}\}$$  \hspace{1cm} (20)

where $\beta$ is the sharpness value. The differentiable function approaches a discontinuous step function at the limit of $\beta \to \infty$. To improve convergence behaviour, a parameter iteration process starting from a small $\beta$ value is applied. $\eta \in [0, 1]$ is the threshold value. It can control the length scale by changing the threshold value (Jun, Clausen, and Sigmund 2017); however, this paper doesn't focus on length scale control, and $\eta = 0.5$ is set for all projection filters.
2.5 Topology optimization model

In this paper, the topology optimization problem for maximizing the nature frequency considering the Hybrid deposit path (HDP) pattern is explored. The topology optimization model is formulated as follows:

\[
\begin{align*}
\text{find : } & \mu, \theta_p \\
\text{max : } & \omega_k = \left(\frac{\omega^2_k}{\omega^2_k + M\mu_k}\right)^{\frac{1}{2}} \\
\text{subjectto : } & (K - \omega^2_k M)u_k = 0 \\
& g(\rho) = \left(\sum_{i=1}^{\alpha} \rho_i v_i\right) - 1 \leq 0 \\
& \mu_e \in [10^{-3}, 1], \forall e; \theta_p' \in [-2\pi, 2\pi]
\end{align*}
\]

(21)

where \(\mu, \theta_p\) are the two sets of design variables.

The objective function \(\omega_k\) is the \(k^{th}\) natural frequency. \(g(\rho)\) is the global volume constraint, \(\rho_e\) and \(v_e\) denotes the relative density and volume (constant) of element respectively. \(V^v\) denotes the predefined volume fraction limit. The former imposes a lower bound of \(10^{-3}\) on element density to avoid singularity issues in finite element stiffness matrix \(K\) and mass matrix \(M\).

Simple bounds on \(\theta_p\) of \(\pm 2\pi\) allow for angle rotation into the optimal orientation without any restriction.

3. Sensitivity analysis

This topology optimization is solved based on sensitivities using the method of moving asymptotes (MMA). In the following, the sensitivities of the objective function and constraint are derived.

3.1. Sensitivity analysis for the objective function

Considering that \(\theta_p\) is independent on the of the design variable \(\mu\), the sensitivity of the objective function \(\partial \omega_k / \partial \mu_e\) could be derived with the chain rule and product rule in the following form.

\[
\frac{\partial \omega_k}{\partial \mu_e} = \frac{1}{2\omega_k \omega^2_k M \mu_k} \left(2\omega^2_k \mathbf{K} - \omega^2_k M\right) u_k + \omega^2_k \left(\frac{\partial \mathbf{K}}{\partial \mu} - \omega^2_k \frac{\partial M}{\partial \mu_e} \right) u_k
\]

(22)

Substitute (2), (5) into (22):

\[
\frac{\partial \omega_k}{\partial \mu_e} = \frac{1}{2\omega_k \omega^2_k M \mu_k} \left[u_k^T \left(\frac{\partial \mathbf{K}}{\partial \mu} - \omega^2_k \frac{\partial M}{\partial \mu_e} \right) u_k\right]
\]

(23)

Recalling (3), the derivatives of the global stiffness \(\mathbf{K}\) and the global mass matrix \(\mathbf{M}\) with respect to the design variable \(\mu\) can be calculated as follows:

\[
\frac{\partial \mathbf{M}}{\partial \mu_e} = \sum_{i=1}^{\alpha} \mathbf{N}^T \frac{\partial \rho_i}{\partial \mu_e} \mathbf{N}_d \Omega
\]

\[
\frac{\partial \mathbf{K}}{\partial \mu_e} = \sum_{i=1}^{\alpha} \mathbf{B}^T \frac{\partial \mathbf{D}_i}{\partial \mu_e} \mathbf{B} d \Omega
\]

(24)

where \(\frac{\partial \rho_i}{\partial \mu_e}\) and \(\frac{\partial \mathbf{D}_i}{\partial \mu_e}\) can be written as follows:

\[
\frac{\partial \rho_i}{\partial \mu_e} = m_e \frac{\partial \rho}{\partial \mu_e} + m_e \frac{\partial \tau_i}{\partial \mu_e} - m_e \left(\frac{\partial \rho}{\partial \mu_e} + \frac{\partial \tau_i}{\partial \mu_e}\right)
\]

(25)

\[
\frac{\partial \mathbf{D}_i}{\partial \mu_e} = \mathbf{D}_e \mathbf{P}(\rho_i)^{\rho_i-1} \frac{\partial \rho_i}{\partial \mu_e} + \mathbf{D}_e \mathbf{P}(\tau_i)^{\rho_i-1} \frac{\partial \tau_i}{\partial \mu_e} + \mathbf{D}_e \mathbf{T}_i \mathbf{T}_i^T \mathbf{D}_e
\]

(26)

Here, the derivations of \(\frac{\partial \rho_i}{\partial \mu_e}\) and \(\frac{\partial \tau_i}{\partial \mu_e}\) could be found in (Clausen, Aage, and Sigmund 2015), and the derivation of the term \(\frac{\partial \mathbf{D}_i}{\partial \mu_e}\) is presented as follows:

\[
\frac{\partial \mathbf{D}_i}{\partial \mu_e} = \frac{\partial \mathbf{D}_i}{\partial \theta_p} \frac{\partial \theta_p}{\partial \mu_e}
\]

(27)

Recalling (10), \(\mathbf{D}_0\) is a constant matrix, using the chain rule.

\[
\frac{\partial \mathbf{D}_i}{\partial \theta_p} = \frac{\partial \mathbf{T}(\theta_p)}{\partial \theta_p} \mathbf{D}_0 (\mathbf{T}(\theta_p))^T + \mathbf{T}(\theta_p) \mathbf{D}_0 (\mathbf{T}(\theta_p))^T
\]

(28)

where

\[
\frac{\partial \mathbf{T}(\theta_p)}{\partial \theta_p} = \begin{bmatrix}
2\sin \theta \cos \theta & \sin 2\theta & \sin 2\theta & \cos 2\theta \\
\sin 2\theta & \cos 2\theta & \cos 2\theta & \sin 2\theta \\
\cos 2\theta & \sin 2\theta & \cos 2\theta & \cos 2\theta \\
\cos 2\theta & \cos 2\theta & \sin 2\theta & \sin 2\theta
\end{bmatrix}
\]

(29)

Recalling (8), \(\frac{\partial \tau_i}{\partial \mu_e}\) is obtained through:

\[
\frac{\partial \tau_i}{\partial \mu_e} = \frac{1}{1 + \left(\frac{\partial \phi}{\partial \mu_e} \frac{\partial \phi}{\partial \mu_e}\right)^2 + \left(\frac{\partial \phi}{\partial \mu_e} \frac{\partial \phi}{\partial \mu_e}\right)^2}
\]

(30)

where

\[
\frac{\partial \phi}{\partial \mu_e} = \frac{\partial \phi}{\partial \theta_p} \frac{\partial \theta_p}{\partial \mu_e}
\]

(31)
To simplify $\partial \hat{\phi}/(\partial y \partial \hat{\phi})$, a new variable $\hat{\phi}_N$ is introduced to this derivation, wherein $\hat{\phi}_N$ is the nodal variable vector related to the elemental variable $\hat{\phi}$. The relationship between $\hat{\phi}$ and $\hat{\phi}_N$ is

$$\hat{\phi} = N^T \hat{\phi}_N, \hat{\phi}_N = T_F \hat{\phi} \tag{32}$$

where $N^T$ is the shape function vector, $T_F$ is the matrix which maps the elemental value to the nodal values. Hence,

$$\frac{\partial \hat{\phi}_e}{\partial \hat{\phi}_e} = \frac{\partial (N^T \hat{\phi}_N)}{\partial \hat{\phi}_e} = B_x^T \frac{\partial \hat{\phi}_N}{\partial \hat{\phi}_e} = B_x^T T_F \tag{33}$$

and

$$\frac{\partial \hat{\phi}_e}{\partial \mu_e} = B_x^T T_F \frac{\partial \phi_i}{\partial \mu_e} \tag{34}$$

The same derivation is applied to $\partial \hat{\phi}_i/(\partial x \partial \phi_i)$, and then, we have:

$$\frac{\partial \theta_i}{\partial \mu_e} = \frac{1}{1 + \left(\frac{\partial \hat{\phi}_i}{\partial \mu_e}\right)^2} \left(\frac{\partial \hat{\phi}_i}{\partial \mu_e} B_x^T T_F - \frac{\partial \hat{\phi}_i}{\partial \mu_e} B_y^T T_F \frac{\partial \phi_i}{\partial \mu_e}\right) \tag{35}$$

Similarly, the independence between $\theta_i$ and $\mu$ leads to:

$$\frac{\partial \omega_k}{\partial \theta_i} = 2 \omega_k \left[u^*_k \left(\frac{\partial K}{\partial \theta_i}\right) u_k\right] \tag{36}$$

$$\frac{\partial K}{\partial \theta_i} = \sum_{j=1}^{NE} B^T \frac{\partial D_i}{\partial \theta_i} B d\Omega \tag{37}$$

where,

$$\frac{\partial D_i}{\partial \theta_i} = (\phi_i)^r \frac{\partial D_i}{\partial \theta_i} - (\phi_i)^t \frac{\partial D_i}{\partial \theta_i} \tag{38}$$

The derivation of $\partial D_i/\partial \theta_i$ resembles (28).

### 3.2. Sensitivity analysis for the volume constraint

The sensitivity of the volume constraint could be written as:

$$\frac{\partial g}{\partial x_e} = \left[\sum_{i=1}^{NE} \left(\frac{\partial D_i}{\partial x_e} N_i\right)\right] / V^*, x \in \{\mu, \theta_i\} \tag{39}$$

The irrelevance between density $\rho$ and raster angle $\theta_i$ leads to:

$$\frac{\partial \rho_i}{\partial \theta_i} = 0 \tag{40}$$

The specific derivation of $\partial \rho_i/\partial \mu_e$ was introduced in (25).

### 3.3. Multiple eigenfrequencies

If the nature frequencies satisfy $\omega_{k1} = \ldots = \omega_{k2} = \tilde{\omega}$, they are called multiple eigenfrequencies or repeated eigenfrequencies. In this case, density sensitivity cannot be calculated straightforwardly from (23) because of the lack of usual differentiability properties of the subspace spanned by the eigenvectors associated with the multiple eigenfrequency. One of the efficient methods has been proposed by Seyranian et al. (Seyranian, Lund, and Olhoff 1994). At first, a generalized gradient matrix $D$ is constructed in the following form (Quhao et al. 2021):

$$D_{ij} = u^*_j \left(\frac{\partial K}{\partial \mu_e} - \tilde{\omega}^2 \frac{\partial M}{\partial \mu_e}\right) u_i, i, j = k1, \ldots, k2 \tag{41}$$

where $u_i$ and $u_j$ are the eigenvectors associated with repeated eigenfrequencies. Then, a sub-eigenvalue problem:

$$\text{det}(D - \Lambda I) = 0 \tag{42}$$

can be solved for obtaining the eigenvalues $\Lambda = [\lambda_{k1}, \ldots, \lambda_{k2}]$, which represent the sensitivities of eigen frequencies with respect to design variables:

$$\left[\frac{\partial \omega_{k1}}{\partial \mu_e}, \ldots, \frac{\partial \omega_{k2}}{\partial \mu_e}\right] = [\Lambda_{k1}, \ldots, \Lambda_{k2}] \tag{43}$$

### 4. Numerical implementation

In this section, the proposed method will be validated with several classical 2D benchmark cases. Four-node quadrilateral elements are adopted in all numerical cases. The following parameters are the same for all examples. A penalization factor $\gamma = 3$ is used in stiffness interpolation for $\varphi$ and $\tau$; the projection threshold is set to be 0.5 for $\eta_1$ and $\eta_2$. The coating sharpness factor $\beta_2$ is initialized with $\beta_2 = 4$ to get a relatively sharp coating from the first iteration and doubled for every 50th iteration. The sharpness factor for the substrate domain starts from $\beta_1 = 1$, and updates the same manner with $\beta_2$. 
The materials are subject to plane stress conditions. The materials of the contour and substrate are the same. Solid materials employ Young’s modulus of $E_x = 2.0 \times 10^7 \text{Pa}$ in the raster direction and $E_y = 0.5 \times 10^7 \text{Pa}$ in the transverse direction. In addition, Poisson’s ratio is 0.4, and the shear modulus is $G_{xy} = 0.35 \times 10^7 \text{Pa}$. Density $m_\varphi = m_\tau = 1\text{kg/m}^3$ is used in all cases. The substrate area is assumed to be fully infilled, and the raster direction is defined positively in the counter-clockwise direction, as shown in Figure 3.

Note that the maximum natural frequency problem will have a trivial solution (Bendsoe and Sigmund 2003): one can, in principle, obtain an infinite natural frequency by removing the entire structure. Therefore, this kind of problem is often used in ‘reinforcement’ problems.
where some areas are fixed to be solid. Specifically, non-structural masses will be placed at points inside the design domain, and the specific locations and magnitudes of the non-structural masses will be elaborated case by case. The whole process of the proposed method is shown in Figure 4.

### 4.1 Short cantilever problem

The first case investigates the topology optimization of the short cantilever problem with length \( L = 1 \text{m} \) and height \( H = 2 \text{m} \). The left edge is clamped, and a concentrated lump mass of magnitude 0.4 kg (50% of the distributable mass, assuming the thickness is 1 m) is placed at the center of the right edge as Figure 5 shows. The design domain is meshed with 100 \( \times \) 200 four-node quadrilateral elements. The objective function is to maximize the fundamental frequency under the global volume fraction constraint of 40%. The raster direction for the substrate material \( \theta_{\rho} \) is initialized with \( \frac{\pi}{2} \). The density variable of all elements is initialized as 0.4. The filter radii in this case are \( R_1 = 10, R_2 = 5 \).

Three cases with different but fixed substrate raster directions (0°, 45°, and 90°) are given in Figure 6. It is noticeable that the optimized structure is no symmetric when the fixed raster angle is 45°. The reason is straightforward that for the lower part, the principal stress direction is close to the direction showing larger material stiffness, so less materials are required, and in contrast, more materials are required for the upper half to balance the overall stiffness distribution since the principal stress direction and the printing longitudinal direction are mismatched.

Figure 7a) shows the optimized structure considering the optimal raster direction \( \theta_{\rho} \) of the substrate. The fundamental frequency of the optimized structure (1998.9 Hz) is higher than that shown in Figure 6 (1736.8 Hz; 1643.7 Hz; 1800.4 Hz), and close to that of \( \theta_{\rho} = 45^\circ \).

Figure 7b) presents the convergence history. The raster direction \( \theta_{\rho} \) changes sharply within the early iterations and gradually converges to 42.4° at around the 200th iteration. The frequency keeps increasing and finally approaches a fixed value. The sharp stair-like oscillations in the nature frequency history curve are

![Figure 5. Sketch of short cantilever.](image)

![Figure 6. The optimized results with different fixed raster directions](image)
caused by the update of sharpness factor $\beta$ for every 50th iteration. Each time the $\beta$ doubles, the PDE filter makes the density field closer to the black-and-white designs. When the result is almost black and white designs, it gradually becomes stable and finally converges. Generally speaking, the proposed method has shown good convergence behavior. Figure 7c) demonstrates the optimized raster direction distribution.

Figure 8 demonstrates the optimized results without considering the HDP pattern. The material properties and parameters are set the same as previous, and the raster direction is set to $0^\circ$. Obviously, its fundamental frequency is the lowest in all the results.

The first three natural frequencies of the optimized results for the different cases are listed in Table 1. The fundamental frequency $\omega_1$ in the case with optimized $\theta_p = 42.4^\circ$ is higher than other cases. However, this advantage does not keep for the higher-order frequencies, the natural frequencies of $\theta_p = 0^\circ$, which has the lowest first-order natural frequency, becomes the highest for the second and third orders.

4.2 Double-clamped structure

The double-clamped structure problem is conducted to further verify the proposed method, and its boundary condition is presented in Figure 9. The structural sizes are defined by $L = 6m$ and $H = 1m$, which is clamped at both ends. The design domain is discretized by $420 \times 70 = 29400$ four-node quadrilateral elements. The maximum material volume ratio is set to be 0.5, and a concentrated lump mass of magnitude 1.5 kg (50% of the distributable mass, assuming

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{(a) The optimized results with optimal raster direction $\theta_p = 42.4^\circ$, $\omega_1 = 1800.4Hz$ (b) The iteration process (c) Raster direction of optimal structure.}
\end{figure}
domain is different for different shapes; therefore, putting the substrate raster directions into optimization is necessary.

The optimized results with raster direction optimization and its convergence history are given in Figure 11. The optimization result without considering the HDP pattern is shown in Figure 12. The material properties and parameters are set the same as previous, and the raster direction is set to 0°, and its fundamental frequency is the lowest in all the results. Combining the results of Figure 8 and Figure 12, it shows the HDP pattern is not only closer to reality, but its optimization frequency will also be higher.

Table 2 gives the first three natural frequencies of the optimized results from different cases. It could be observed that the optimized structure could achieve the highest first fundamental frequency. However, compared to other designs with fixed substrate raster direction, the optimized design in Figure 11 may not obtain the best performance given other orders of the natural frequency.

### 4.3 Multi-frequency optimization

Although the maximum fundamental frequency of the system is usually used as the objective function, however, in some cases, the frequency optimization may simultaneously consider several lowest fundamental frequencies, since as shown in Table 1 and Table 2, the fundamental frequencies of the optimized structure cannot keep the highest for all the first three orders. Therefore, the objective function can be modified into (Q. Liu, Chan, and Huang 2016):

$$\max \omega_c = w_1\omega_1 + w_2\omega_2 + \ldots + w_n\omega_n$$  \hspace{1cm} (44)

where $n$ represented the order number of the natural frequency, $w_k$ is the prescribed weight factor which defines the relative importance of each fundamental frequency and $\sum_{k=1}^n w_k = 1$. 

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**Figure 8.** The optimized result without considering the HDP pattern (raster direction $\theta = 0^\circ$): $\omega_1 = 1434.3$Hz.

**Figure 9.** Sketch of double-clamped structure.

**Figure 10.** Demonstrates the topology optimization with three different fixed substrate raster directions ($0^\circ$, $45^\circ$, and $90^\circ$). A different phenomenon could be observed in this example compared with the short cantilever problem: the fundamental frequency of the topology optimization with $\theta_\phi = 0^\circ$ is higher than others, which is reasonable since the principal stresses are distributed along the horizontal direction. It is proved that the best raster direction of the substrate
Figure 10. The optimized results with different fixed substrate raster directions (a) $\theta_\phi = 0^\circ$, $\omega_1 = 536.1\text{Hz}$; (b) $\theta_\phi = 45^\circ$, $\omega_1 = 485.1\text{Hz}$; (c) $\theta_\phi = 90^\circ$, $\omega_1 = 441.3\text{Hz}$.

Figure 11. (a) The optimization result with optimized substrate raster direction $\theta_\phi = 0.6^\circ$, $\omega_1 = 536.2\text{Hz}$ (b) The iteration process.
The optimization result of the double-clamped beam structure as introduced in Section 4.2 is shown in Figure 13, wherein the first three natural frequencies are considered and the weight factors of $w_1 = w_2 = w_3 = 1/3$ are assigned while all other parameters keep unchanged.

Table 2 presents the first three natural frequencies. Although the fundamental frequency (230.3 Hz) is lower than that shown in Table 2 (536.2 Hz), the averaged value of the first three natural frequencies ($\omega_o = 2285.3$ Hz) is much higher than those shown in Table 2 (1451.7 Hz; 1359.7 Hz; 1248.3 Hz; 1452.9 Hz for $\theta_\varphi = 0^\circ$; 45°; 90°; 0.6° respectively).

### 4.4 Parameter Independence investigation

In the first two examples, the substrate raster directions both start from $\pi/2$. To show the parameter independence of the proposed method, optimizations with the starting raster directions of 0 and $\pi/4$ are performed, and the optimization results are demonstrated in Figure 14 and Figure 15, respectively.

As can be seen, the final topological structures and raster directions are both close to the optimization result from the initial guess of $\theta_\varphi = \pi/2$. The results indicate that the initial raster direction $\theta_\varphi$ would
perturb the optimization result, but the perturbation could be omitted since the influence is slight (less than 1%).

4.5 The corresponding mode

In this subsection, a case study is presented to explore the influence of structural optimization on the corresponding vibration modes. As shown in Figure 16, the boundary condition is changed while all other optimization setups remain the same as subsection 4.2, i.e. the center areas of

Table 3. The first three natural frequencies (Hz) of the optimization result.

<table>
<thead>
<tr>
<th>ðθpθφ</th>
<th>ðω1</th>
<th>ðω2</th>
<th>ðω3</th>
</tr>
</thead>
<tbody>
<tr>
<td>ðθp = -1.23°</td>
<td>230.3</td>
<td>1344.0</td>
<td>5281.6</td>
</tr>
</tbody>
</table>

Figure 14. (a) The optimization result with finally derived ðθpθφ = 41.2°, ðω1 = 1799.0Hz; (b) The iteration history for the design with starting raster direction of ðθp = 0°.

Figure 15. (a) The optimization result with finally derived ðθpθφ = 41.5°, ðω1 = 1800.1Hz; (b) The iteration history for the design with starting raster direction of ðθp = 45°.

Figure 16. The boundary condition for the 2D beam.
the two ends are fixed while all other settings remain consistent. The optimized structure is presented in Figure 17, wherein the optimized substrate raster direction is $0.1^\circ$, and the final first-order fundamental frequency is 362.9Hz.

The first three vibration modes of optimized structure are given in Figure 18, and the color bar represents the size of strain. Figure 19 and Figure 20 present the first three vibration modes of the optimization results with fixed substrate raster directions of $\theta_p = 45^\circ$ and $\theta_p = 90^\circ$, respectively. All these vibration modes are global, and there appears no local mode vibration. Comparing the above three figures, although the optimized structural shapes from different raster directions are different, the vibration modes are close. The first mode vibrates in the y-direction, the second vibrate in the x-direction, and the third mode works with the torsional vibration. Hence, the change of substrate raster direction does not alter the fundamental vibration modes.

In the end, the computational time is briefly discussed. All the above cases were run on a desktop computer with Intel Core i5-8600 K CPU and 16GB RAM. Taking the case in section 4.1 as an example, the average time spent per iteration was 15 s: the FEM part accounted for
59.13%, the sensitivity analysis part takes 16.45%, the MMA solver takes 20.32%, and the rest accounted for 4.1%. For the other cases, the percentage of each part is similar, but for more elements, such as $420 \times 70$ in section 4.2, the time spent per iteration is up to 21 s.

5. Conclusion

This paper proposed a topology optimization method that considers the HDP pattern-induced anisotropic material properties to maximize the fundamental structural frequency. Two sets of design variables (density variable $\mu$ and the raster direction variable $\theta$ for the substrate domain) are introduced to perform the concurrent structure and process parameter optimization. Several numerical examples are studied in this paper, showing the strong influences and necessities of involving the HDP pattern and anisotropic material properties in optimization. Hence, the concurrent optimization scheme leads to close-to-reality design results than the existing methods that assume isotropic material models and would be instructive to dynamic design problems for additive manufacturing, especially the material extrusion-type processes.

There is still further space to extend the current work. In addition to the HDP pattern, additive manufacturing fits with even more complex deposition path patterns, for instance, the medial axis-based deposition pattern. Modeling and performing sensitivity analysis on other more complex deposition patterns are technically non-trivial and deserve in-depth investigations. Besides, this paper only considered 2D cases due to limited computation power. There are differences for 3D structures that involve one extra dimension of vibration. Therefore, the extension to cover 3D cases will be targeted in our future work.

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