ASSIGNMENT 6

Due date Monday 11th of May

(1) Find a power series expansion for the principle branch of $\log(z)$ around $z = 1$. What is the radius of convergence?
(2) Question 21 page 107 of the textbook
(3) Consider the multifunction $f(z) = (z-1)^{\frac{1}{2}}(z-i)^{\frac{1}{3}}$
   (i) Where are the branch points and what are their orders?
   (ii) Give an example of a simply connected subset of the complex plane on which a branch of $f(z)$ can be defined.
   (iii) How many values does $f(z)$ have at points that are not branch points? List all the possible values at $z = 0$.
   (iv) Choose one of the possible values just listed, and label it p. Sketch a path starting and ending at the origin such that if $f(0)$ is initially chosen to be equal to -1, then as z travels along the path and returns to the origin, $f(z)$ travels along a path from -1 to p. Repeat this for all other possible values of p.
(4) For each function listed below, find all the branch points and singularities (if any). For the case of multivalued functions, give an example of a simply connected subset of the complex plane on which a branch of the multifunction can be defined.
   (i) $f(z) = \frac{1}{e^{\pi z}-1}$
   (ii) $f(z) = (z^4-1)^{\frac{1}{5}}$
   (iii) $f(z) = (z-i)^{\frac{1}{2}}(z-1)^{-1}$
(5) What value does z^i take at $z = -1$ if we start with $1^i = 1$ at $z = 1$ and then let z trace out a path parameterised by $e^{3i\pi t}$, for t in $[0, 1]$?